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Transgression forms and extensions of Chern-Simons gauge theories

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Abstract: A gauge invariant action principle, based on the idea of transgression forms, is proposed. The action extends the Chern-Simons form by the addition of a boundary term that makes the action gauge invariant (and not just quasi-invariant). Interpreting the spacetime manifold as cobordant to another one, the duplication of gauge fields in spacetime is avoided. The advantages of this approach are particularly noticeable for the gravitation theory described by a Chern-Simons lagrangian for the AdS group, in which case the action is regularized and finite for black hole geometries in diverse situations. Black hole thermodynamics is correctly reproduced using either a background field approach or a background-independent setting, even in cases with asymptotically nontrivial topologies. It is shown that the energy found from the thermodynamic analysis agrees with the surface integral obtained by direct application of Noether's theorem.

Contents

1. Introduction

Unlike the theories for the other three known fundamental interactions, General Relativity (GR), the currently accepted theory of gravitation described by the Einstein-Hilbert action, is neither a gauge theory for the Poincaré nor the (A)dS groups [1]. The reason is that while the spin connection can be regarded as a gauge potential for the Lorentz group, the vielbein does not transform as a connection for translations or (A)dS boosts.

Since the geometry is dynamically determined, the construction of a gauge theory of gravity would require an action without reference to a fixed space-time background. This requirement rules out actions of the Yang-Mills type, which requires a background metric. It turns out that the only possibility of a Lagrangian for gravity in terms of a connection without extra fields, is given by the Chern-Simons (**CS**) form for some space-time group (like the de Sitter, anti de Sitter or Poincaré groups)[1, 2].

A CS gauge theory is one whose Lagrangian is given by the CS form for a gauge group. These theories have been studied in a variety of contexts (see for instance refs.[3, 4, 5]). In particular, CS gravities and supergravities are defined by a space-time gauge group or one of its supersymmetric extensions. These theories were introduced in refs. [6, 7, 8] for three-dimensional space-times. It was noticed that General Relativity in 2+1 dimensions is a CS theory for the Poincaré group, ISO(2,1), a fact that was exploited by Witten to show that the theory is exactly solvable at the quantum level[8]. There also exist gravity theories in higher odd dimensions described in terms of CS actions [9, 10]. For negative cosmological constant, the local supersymmetric extension in five dimensions was given in [11], and for higher odd dimensions in [12, 13]. In the absence of cosmological constant the corresponding supergravity theories were constructed in [14, 15, 16].

In [13] it was also suggested that the low energy limit of M-theory [17, 18, 19, 20] may be a CS theory with gauge group $OSp(1|32)$, a proposal also explored in [21, 22, 23].

For dimensions $d > 3$, CS gravity theories are not equivalent to GR. The question of the relationship between CS and GR in diverse dimensions has been studied in refs.[9, 2, 14, 12, 21]. Recently a new approach to this problem, and to the related one of finding a "non degenerate vacuum", has been discussed in ref. [16] for the eleven-dimensional supergravity invariant under the M-algebra.

In this work we consider theories based on *transgression forms*, which are generalizations extending CS forms by the inclusion of a second gauge field [24, 25, 26, 27, 28, 29]. Conversely, CS forms can be thought of as transgression forms with one of the gauge fields set equal to zero. The second gauge field in the transgression form can be interpreted either as a fixed, non-dynamical background, or as a dynamical

[1] For a discussion on this point, including references, see e. g., [1, 2]

field on the same footing as the first one. In the second case, it is possible to conceive both fields as defined on different manifolds with a common boundary, thus eliminating possible ambiguities in the physical interpretation. This is the point of view we advocate here.

Transgression forms can be used to define actions for physical systems that give rise to well defined conserved charges [30, 31], and in the construction of actions for extended objects [32, 33]. More recently, transgression forms have also been used in [34, 35, 36]. Transgression forms in field theory were also the central topic of ref.[37].

Using transgression forms to construct the action has several advantages:

(i) The transgression form singles out a unique boundary term for the action principle, allowing both background-dependent and background-independent formulations of the same system. In particular, in the background-independent approach, as shown in Ref. [31], it provides a well defined variational principle for a wide set of boundary conditions.

(ii) In the case of gravitation, the boundary terms introduced by the transgression allow to regularize the action for black hole configurations in diverse situations. Thus, black hole thermodynamics is obtained using either a background field approach or a background-independent setting, even in cases with asymptotically nontrivial topologies. The results agree with the ones computed by hamiltonian methods [38, 39, 40].

(iii) Conserved charges can be constructed as surface integrals through the Noether method. The energy obtained in this way agrees with the result found from thermodinamics.

The plan of this work is as follows. Section 2 is devoted to the construction of lagrangians as transgression forms in which the two connection fields A and $\overline{A}$ coexist in the same spacetime manifold, which is sufficient to compute some quantities of physical interest. In Section 3 this construction is applied to gravity with negative cosmological constant, in order to make contact with the background-independent approach of [31]. In section 4, the interpretation of Section 2 is revisited and a proposal is advanced where the two fields A and $\overline{A}$ are regarded as having support in two distinct manifolds M and $\overline{M}$ with a common boundary. In Section 5, the new setting is applied to the spacetime geometry, in particular for black holes of various dimensions and diferent topologies verifying points (ii) and (iii) above. Section 6 contains the discussion and comments, while reviewed material and some detailed calculations are contained in the appendices.

2. Transgression forms as Lagrangians

A Chern-Simons form $\mathcal{C}_{2n+1}(A)$ is a differential form defined for a connection A, whose exterior derivative yields a Chern class. Although the Chern classes are gauge invariant, the CS forms are not; under gauge transformations they change by a closed

form. A transgression form $\mathcal{T}_{2n+1}$ on the other hand, is an *invariant* differential form whose exterior derivative is the *difference* of two Chern classes. It generalizes the CS form with the additional advantage that it is gauge invariant. The price to pay is that it is a function of two connections A and $\overline{A}$. In fact, a transgression form can be written as the difference of two CS forms plus an exact form,

$$\mathcal{T}_{2n+1} = \mathcal{C}_{2n+1}(A) - \mathcal{C}_{2n+1}(\overline{A}) - dB_{2n}\left(A, \overline{A}\right). \tag{2.1}$$

It can be written as (see e.g., [29]),

$$\mathcal{T}_{2n+1}\left(A, \overline{A}\right) = (n+1)\int_0^1 dt \ \ < \Delta A F_t^n > , \tag{2.2}$$

where[2]

$$\begin{aligned} A_t &= tA + (1-t)\overline{A} \\ &= \overline{A} + t\Delta A \ , \end{aligned} \tag{2.3}$$

is a connection that interpolates between the two independent gauge potentials A and $\overline{A}$. The Lie algebra-valued one-forms $A = A_\mu^I T_I \ dx^\mu$ and $\overline{A} = \overline{A}_\mu^I T_I \ dx^\mu$ are connections under gauge transformations, T_I are the generators, and $< \cdots >$ stands for a symmetrized invariant trace in the Lie algebra (see Appendix A). The corresponding curvature is

$$\begin{aligned} F_t &= dA_t + A_t^2 \\ &= tF + (1-t)\overline{F} - t(1-t)(\Delta A)^2 \ , \end{aligned} \tag{2.4}$$

and the explicit expression for the $2n$-form C_{2n} is

$$B_{2n} = -n(n+1)\int_0^1 ds \int_0^1 dt \ s \ \ < A_t \Delta A \ F_{st}^{n-1} > \tag{2.5}$$

where $F_{st} = sF_t + s(s-1)A_t^2$. Hence, the role of the surface term B_{2n} is to cancel the variation of the bulk terms $\mathcal{C}_{2n+1}$, which change by a closed form under a gauge transformation. The pure CS density is recovered setting $\overline{A} = 0$.

Transgression forms can be used as a field theory Lagrangians for gauge fields A and $\overline{A}$, where B_{2n} is the interaction term which is only defined at the boundary.

Conserved charges written as surface integrals for CS theories have been obtained using different approaches in Refs. [41, 42]. Since the transgression form (2.2) is manifestly invariant under diffeomorphisms and gauge transformations where both A and $\overline{A}$ transform as connections simultaneously, the corresponding conserved charges can be simply written as surface integrals by direct application of Noether's theorem.

[2] Here wedge product between forms is assumed.

Assuming suitable asymptotic conditions for the fields, one obtains the conserved charges associated with an asymptotic Killing vector ξ as

$$Q(\xi) = n(n+1) \int_{\partial\Sigma} \int_0^1 dt < \Delta A F_t^{n-1} I_\xi A_t > , \tag{2.6}$$

where $\partial\Sigma$ is the boundary of the spatial section Σ. Analogously, for an asymptotically covariantly constant Lie algebra valued parameter $\lambda = \lambda^I T_I$, $D\lambda = 0$, the charges correspond to

$$Q(\lambda) = n(n+1) \int_{\partial\Sigma} \int_0^1 dt < \Delta A F_t^{n-1} \lambda > \ . \tag{2.7}$$

The explicit derivation of (2.6) and (2.7) is simple [37, 43], and is reproduced here in Appendix B.

As explicit examples, the expression for the transgression form in three dimensions is

$$\mathcal{T}_3 = \mathcal{C}_3(A) - \mathcal{C}_3(\overline{A}) - d < A\overline{A} > , \tag{2.8}$$

where $\mathcal{C}_3(A)$ stands for the CS form

$$\mathcal{C}_3 =< AF - \frac{1}{3}A^3 > \ . \tag{2.9}$$

Similarly, in five dimensions the transgression form turns out to be

$$\mathcal{T}_5(A,\overline{A}) = \mathcal{C}_5(A) - \mathcal{C}_5(\overline{A}) - dB_4(A,\overline{A}) , \tag{2.10}$$

where the five-dimensional CS Lagrangian reads

$$\mathcal{C}_5 =< AF^2 - \frac{1}{2}A^3F + \frac{1}{10}A^5 > , \tag{2.11}$$

and the boundary term is

$$B_4(A,\overline{A}) = \frac{1}{2} < (A\overline{A} - \overline{A}A)(F+\overline{F}) + \overline{A}A^3 + \overline{A}^3 A + \frac{1}{2}A\overline{A}A\overline{A} > \ . \tag{2.12}$$

3. Finite action for gravity for the AdS group

Following the same basic principles of General Relativity in higher dimensions allows for a wide class of gravity theories. The generalization of the Einstein-Hilbert Lagrangian for any dimension d are the so-called the Lovelock Lagrangians [44],[3]

$$L_{Lovelock} = \kappa \int_M \sum_{p=0}^{n} \alpha_p L^{(p)} , \tag{3.1}$$

where $L^{(p)}$ are the dimensional continuations of Euler densities from lower dimensions

$$L^{(p)} = \epsilon_{a_1 \cdots a_d} R^{a_1 a_2} \cdots R^{a_{2p-1} a_{2p}} \ e^{a_{2p+1}} \cdots e^{a_d} \ .$$

Here e^a is the vielbein one-form, and $R^{ab} = d\omega^{ab} + \omega^a_{\ c}\omega^{cb}$ is the curvature two-form.

[3]Latin indices a, b run from $0, ..., d-1$.

3.1 Chern-Simons gravity

For $d = 2n + 1$ and the special choice of coefficients

$$\alpha_p = \frac{l^{2(p-n)}}{d-2p}\begin{pmatrix} n \\ p \end{pmatrix} \ , \tag{3.2}$$

the action corresponds to CS form for the AdS group [9], up to a boundary term. It is useful to rewrite the series (3.1) with the choice (3.2) as an integral over a continuous parameter $t \in [0, 1]$,

$$L_{CS}(\omega^{ab}, e^a) = \kappa \int_0^1 dt \ \epsilon_{a_1 \cdots a_{2n+1}} R_t^{a_1 a_2} \cdots R_t^{a_{2n-1}a_{2n}} \ e^{a_{2n+1}} \ , \tag{3.3}$$

with

$$R_t^{ab} \equiv R^{ab} + t^2 e^a e^b \ ,$$

where the AdS radius l has been set equal to one. Hereafter we choose $\kappa = [2(d-2)!\Omega_{d-2}G]^{-1}$, where Ω_{d-2} is the volume of the sphere S^{d-2}, and G is the d-dimensional 'Newton constant'.

The explicit link between this action and a CS form can be seen as follows. The connection for the AdS group $SO(d-1, 2)$ reads

$$A = \frac{1}{2}\omega^{ab} J_{ab} + e^a P_a \ , \tag{3.4}$$

where J_{ab}, and P_a stand for the generators of Lorentz rotations and AdS boosts, respectively. Here e^a is identified with the vielbein and ω^{ab} with the spin connection. The corresponding curvature $F = dA + A^2$ is given by

$$F = \frac{1}{2}\left(R^{ab} + e^a e^b\right) J_{ab} + T^a P_a \ ,$$

where $T^a = De^a$ is the torsion two-form.

The AdS group admits an invariant tensor yielding the symmetric trace

$$< J_{a_1 a_2} ... J_{a_{2n-1}a_{2n}} P_{a_{2n+1}} >= \kappa \frac{2^n}{(n+1)} \epsilon_{a_1 ... a_{2n+1}} \ , \tag{3.5}$$

and it is simple to check that the Lagrangian in Eq.(3.3)

$$L_{CS}(\omega^{ab}, e^a) = \kappa \int_0^1 dt \ \epsilon \left(R_t\right)^n \ e \ ,$$

satisfies[4]

$$dL_{CS} = \kappa\epsilon(R + e^2)^n T =< F^{n+1} > \ , \tag{3.6}$$

and hence, the Lagrangian is a CS form, up to an exact form.

[4]Here, for simplicity we have omitted the indices, which are assumed to be contracted in canonical order.

3.2 Transgression

The explicit expression for the transgression form (2.2) for the AdS group is obtained defining $\overline{A} = \frac{1}{2}\overline{\omega}^{ab} J_{ab} + \overline{e}^a P_a$ (see Appendix C), and is written in terms of (3.3) as

$$\mathcal{T}_{2n+1} = L_{CS}(\omega, e) - L_{CS}(\overline{\omega}, \overline{e}) - dB_{2n} \ . \tag{3.7}$$

Since the transgression form is invariant by construction under local Lorentz transformations, it cannot depend separately on ω and $\overline{\omega}$, but only through the combination $\Delta\omega = \omega - \overline{\omega}$, which transforms as a tensor. Indeed, the boundary term is given by

$$B_{2n} = \kappa n \int_0^1 dt \int_0^1 ds \ \epsilon \ \Delta\omega \ e_t \left[tR + (1-t)\overline{R} - t(1-t)(\Delta\omega)^2 + s^2 e_t^2 \right]^{n-1} \tag{3.8}$$

where, from Eq. (2.3), $e_t = te + (1-t)\overline{e}$.

Note that the Lorentz invariance of (3.7) is manifest since the curvatures, vielbeins and $\Delta\omega$ are Lorentz tensors. However these are not tensors under AdS boosts and therefore, although the full local AdS invariance is ensured by construction, this invariance is not manifest in (3.7).

An explicit example of (3.7) and (3.8), in $d = 2{+}1$ dimensions is the transgression form

$$\mathcal{T}_3 = \kappa\epsilon_{abc}(R^{ab}e^c + \frac{1}{3}e^a e^b e^c) - \kappa\epsilon_{abc}(\overline{R}^{ab}\overline{e}^c + \frac{1}{3}\overline{e}^a\overline{e}^b\overline{e}^c) - \kappa\frac{1}{2}\epsilon_{abc} d[\Delta\omega^{ab}(e^c + \overline{e}^c)] \ , \tag{3.9}$$

which in a more compact notation reads

$$\mathcal{T}_3 = \kappa\epsilon(Re + \frac{1}{3}e^3) - \kappa\epsilon(\overline{R}\overline{e} + \frac{1}{3}\overline{e}^3) - \frac{1}{2}\kappa\epsilon d[\Delta\omega(e + \overline{e})] \ . \tag{3.10}$$

Analogously, the transgression form in $d = 4 + 1$ is

$$\begin{aligned}
\mathcal{T}_5 = {} & \kappa\epsilon(R^2 e + \frac{2}{3}Re^3 + \frac{1}{5}e^5) - \kappa\epsilon(\overline{R}^2\overline{e} + \frac{2}{3}\overline{R}\overline{e}^3 + \frac{1}{5}\overline{e}^5) \\
& -\frac{1}{3}\kappa\epsilon \ d[\Delta\omega(e + \overline{e})(R - \frac{1}{4}(\Delta\omega)^2 + \frac{1}{2}e^2) + \Delta\omega(e + \overline{e})(\overline{R} - \frac{1}{4}(\Delta\omega)^2 + \frac{1}{2}\overline{e}^2) \qquad (3.11) \\
& + \Delta\omega Re + \Delta\omega\overline{R}\overline{e}] \ . \qquad (3.12)
\end{aligned}$$

In what follows, it is shown that the transgression form provides the suitable boundary terms which yield well defined and finite action principles adapted to different situations. Remarkably, regularized action principles using background fields, as well as finite background-independent actions can be obtained as particular cases within a unique framework. The thermodynamics of black holes is then reproduced in both settings even in cases where the horizon manifold has a nontrivial topology.

3.3 Background-independent action and conserved charges

A finite action principle that is background independent, must depend only on the intrinsic geometric quantities, as the metric and the curvature, as well as on the extrinsic curvature of the manifold at the boundary. This means that apart from the vielbein and the curvature, the boundary term could depend only on the second fundamental form, which is defined as

$$\theta^{ab} = \omega^{ab} - \bar{\omega}^{ab} ,$$

where $\bar{\omega}^{ab}$ is defined only at the boundary. Hence, $\bar{\omega}^{ab}$ can be naturally identified with the spin connection of an auxiliary manifold $\bar{M}$ which is cobordant with M, (i.e. $\partial M = \partial\bar{M}$), and is endowed with a product metric which matches the metric of M at the boundary.

As a consequence, the required objects are the vielbein e^a, the spin connection ω^{ab}, as well as the auxiliary spin connection $\bar{\omega}^{ab}$ chosen as described above. Such an action principle can then be obtained through a transgression form (2.2) with

$$\begin{aligned} A &= \frac{1}{2}\omega^{ab}J_{ab} + e^a P_a , \\ \overline{A} &= \frac{1}{2}\bar{\omega}^{ab}J_{ab} . \end{aligned} \tag{3.13}$$

It is worthwhile to point out that, since transgression form is invariant by construction under local Lorentz transformations, it does not depend separately on ω and $\bar{\omega}$, but only through the combination $\Delta\omega^{ab} = \omega - \bar{\omega} = \theta^{ab}$, which transforms as a tensor.

As announced in Ref. [31], the required action principle is then recovered by means of Eqs. (3.7) and (3.8), so that the action becomes

$$I = \int_M \mathcal{T}_{2n+1} ,$$

with $\bar{e}^a = 0$. In concrete, replacing $\bar{e}^a = 0$ annihilates the second bulk term in Eq. (3.7). Making the same replacement in (3.8), using the Gauss-Codazzi equation $R^{ab} = \bar{R}^{ab} + (\theta^2)^{ab}$ for the relevant components at the boundary, and changing $st \to s$, the boundary term turns out to be

$$B_{2n} = \kappa n \int_0^1 dt \int_0^t ds \; \epsilon \; \theta \; e \left(\bar{R} + t^2\theta^2 + s^2 e^2\right)^{n-1} ,$$

in agreement with [31]. It was shown that this boundary term is sufficient to render the action finite for asymptotically locally AdS solutions. Furthermore, the Euclidean continuation of the action correctly describes the black hole thermodinamics in the canonical ensemble even in cases with asymptotically nontrivial topology.

Following the same proceduce, the conserved charges written as surface integrals obtained in [31]

$$Q(\xi) = \kappa n \int_{\partial\Sigma} \int_0^1 dt \; t \; \epsilon \left(I_\xi \theta e + \theta I_\xi e\right) \left(\bar{R} + t^2\theta^2 + t^2 e^2\right)^{n-1} ,$$

are recovered from Eq. (2.6). It is worth mentioning that although the black hole mass can be computed from two radically different approaches, namely form thermodynamics or form the surface integrals, both results completely agree, including the for the zero point (or Casimir) energy, which corresponds to the mass of the locally AdS solutions.

4. Reinterpretation of the theory

The complete definition of the theory involves both the action principle and suitable boundary conditions. For the action principle to be well defined the action must have an extremum for solutions of the field equations satisfying the boundary conditions.

The action could be taken to be just the integral on a single manifold M of the transgression form. This field theory would describe two self-interacting fields A and $\overline{A}$, which only interact with each other at the boundary. This is a rather strange state of affairs: there is a duplicity of identical dynamical fields which coexist in the spacetime M, but don't affect each other, except by their interaction at the boundary However, since the kinetic term for $\overline{A}$ has the wrong sign, this field would be a *phantom* with an ill-defined propagator.

One way to avoid this conflict is to assume $\overline{A}$ to be a non dynamical background field. This action, however, would be gauge invariant only up to a surface term.

There is a different conceptual framework where the transgression naturally fits in.and which is free from the difficulties mentioned above, i. e., gauge invariance and absence of phantoms. The idea is to conceive the fields A and $\overline{A}$ as defined on cobording manifolds M and $\overline{M}$, respectively, such that $\partial M \equiv \partial \overline{M}$. Then, we propose the following action principle

$$I_{trans} = \int_M \mathcal{C}_{2n+1}(A) - \int_{\overline{M}} \mathcal{C}_{2n+1}(\overline{A}) - \int_{\partial M} B_{2n}\left(A, \overline{A}\right) , \tag{4.1}$$

which describes two Chern-Simons systems interacting only at their common boundary.

Some remarks are in order:

(i) The action of Eq. (4.1) is exactly invariant under gauge transformations since both A and $\overline{A}$ transform as connections in their respective domains, provided the gauge transformation is continuous across ∂M.

(ii) The sign difference between the integrals on M and on $\overline{M}$ can be understood as due to the difference in orientations necessary to match the two manifolds at their common boundary.

(iii) The action (4.1) is not the integral of the transgression form on a single manifold with boundary.

What emerges is a field theory for two subsystems in contact at their common boundary, each described by a CS lagrangian. This description is most natural in

the analysis of black hole thermodynamics below, where the two manifolds M and $\overline{M}$ don't even have the same topology (see, e.g., the construction in [31]).

The variation of the action (4.1) is (see Appendix A)

$$\delta I_{trans} = (n+1)\int_M < F^n \delta A > - \int_{\overline{M}} < \overline{F}^n \delta \overline{A} > + \int_{\partial M} \Theta_{2n} \tag{4.2}$$

The field equations,

$$< F^n T_I > |_M = 0 \ , \ < \overline{F}^n T_I > |_{\overline{M}} = 0 \ \ ,$$

coincide with those of a pure CS theory for two independent connections in the corresponding manifolds M and $\overline{M}$. The action attains an extremum provided the boundary term

$$\Theta_{2n} = -n(n+1)\int_0^1 < \Delta A F_t^{n-1} \delta A_t > \tag{4.3}$$

vanishes on ∂M. A sufficient boundary condition would be, for instance, to require $\Delta A \to 0$ at the boundary, with a fast enough fall-off in the direction normal to the boundary, while F_t remains finite at the boundary so that $\Theta_{2n} = 0$. One may call this case where the connection A approaches a reference field configuration $\overline{A}$ at the boundary, *background dependent*. Alternatively, the approach in which A and $\overline{A}$ are both dynamical fields, can be called *background independent*. Of course, there exist infinitely many other ways to ensure the vanishing of (4.3), in which some components approach a reference connection, while others don't, for example. We shall make use of this possibility in Sect. 3.1 below.

In varying the action, one could also assume that A is dynamical, while $\overline{A}$ is a fixed background which should not be varied. In this case, the second term in the R.H.S. of (4.2) wouldn't exist and only A needs to satisfy the field equations. In any case, $\overline{A}$ could also be taken as a special solution of the field equations, identified as a "vacuum". However this means that the canonical realization of gauge invariance may break down at the boundary.

5. Application to gravity

5.1 Black hole thermodynamics with a reference geometry

The purpose of this subsection is to show that the same transgression form (2.2) also provides an alternative way to obtain a regularized action with a reference background geometry ($\overline{e}^a \neq 0$), which may work in more exotic situations.

The action principle we now consider is

$$I_{trans} = \int_M L_{CS}(\omega, e) - \int_{\overline{M}} L_{CS}(\overline{\omega}, \overline{e}) - \int_{\partial M} B_{2n} \ , \tag{5.1}$$

where L_{CS} is the Lagrangian defined by Eq. (3.3), and B_{2n} is the boundary term (3.8). Here the spin connection ω^{ab} and the vielbein e^a have support only on the manifold M, while $\overline{\omega}^{ab}$ and $\overline{e}^a$ have support only on the cobordant manifold $\overline{M}$.

In what follows it is shown that in the Euclidean continuation, the regularized action (5.1) is finite and gives the correct free energy in the canonical ensemble for black holes, even in cases with nontrivial topology. It is also shown that the energy found from the thermodynamic analysis coincides with the one obtained from the Noether theorem.

5.1.1 Euclidean action and black hole thermodynamics

We consider a family of black holes whose horizons may have a non-spherical topology, labeled by the parameter γ which can take the values $\pm 1, 0$. The line element is [45, 46, 40, 39]

$$ds^2 = -\Delta^2 dt^2 + \frac{1}{\Delta^2} dr^2 + r^2 d\Sigma_{d-2}^2 \ , \tag{5.2}$$

with

$$\Delta^2 = \gamma - \sigma + r^2 \ , \tag{5.3}$$

where $d\Sigma_{d-2}^2$ is the line element of the $(d-2)$-dimensional base manifold of constant curvature proportional to $\gamma = 1, 0, -1$. The horizon is located at $r_+ = \sqrt{\sigma - \gamma}$, and in the euclidean continuation, the manifold for a massive black hole has a radial coordinate that extends over the range $r_+ \leq r < \infty$. The euclidean time period β which determines the temperature is found demanding smoothness of the Euclidean solution at the horizon

$$\beta = T^{-1} = \frac{2\pi}{r_+} \ .$$

For a fixed temperature, in the semiclassical approximation, the Euclidean action is related to the free energy F in the canonical ensemble, $I_E = -\beta F = -\beta E + S$.

Here it is shown that the black hole thermodynamics is reproduced evaluating the solution in euclidean continuation of the action in Eq. (5.1). Thus, we consider A corresponding to a black hole solution of the form (5.2), while $\overline{A}$ is assumed to be a reference configuration given by a suitable solution of the field equations. Since the time period β is fixed for the Euclidean black hole solution, the reference background must be such that $\bar{\beta}$ is arbitrary in order to have a well-defined cobordism between the manifolds M and $\bar{M}$. This requirement is fulfilled for the solution (5.2)) with $r_+ = 0$, i.e., for $\bar{\sigma} = \gamma$, as well as for AdS spacetime which corresponds to $\bar{\sigma} = 0$ in the spherically symmetric case ($\gamma = 1$).

A. Asymptotic spherical symmetry ($\gamma = 1$)

In this case there are two possible reference geometries with arbitrary β, AdS space and the "black hole vacuum" with $\bar{\sigma} = 1$. In both cases the range of the radial coordinate is $0 \leq r < \infty$.

The bulk contributions to the euclidean continuation of the action (5.1) are

$$I_{trans}^{bulk} = \kappa\beta(d-2)!\Omega_{d-2}\int_0^1 ds\ [(\sigma+(s^2-1)r^2)^n + 2nr^2(s^2-1)(\sigma+(s^2-1)r^2)^{n-1}]\Big|_{r=r_+}^{r=\infty}$$
$$-\kappa\beta(d-2)!\Omega_{d-2}\int_0^1 ds\ [(\overline{\sigma}+(s^2-1)r^2)^n + 2nr^2(s^2-1)(\overline{\sigma}+(s^2-1)r^2)^{n-1}]\Big|_{r=0}^{r=\infty} ,$$

where Ω_{d-2} is the volume of the $d-2$-dimensional unit sphere. The boundary term is

$$\begin{aligned}\int_{\partial\mathcal{M}} B_{2n} = \ & -2n\kappa\beta(d-2)!\Omega_{d-2}\lim_{r\to\infty}\int_0^1 dt\int_0^1 ds\{(\Delta-\overline{\Delta})[t\Delta+(1-t)\overline{\Delta}]\times \\ & \times[1-(t\Delta+(1-t)\overline{\Delta})^2+s^2r^2]^{n-1} \\ & +2(n-1)r^2(s^2-1)(\Delta-\overline{\Delta})[t\Delta+(1-t)\overline{\Delta}][1-(t\Delta+(1-t)\overline{\Delta})^2+s^2r^2]^{n-2}\}\end{aligned}$$

Integration on t can be performed with the substitution $u=1-(t\Delta+(1-t)\overline{\Delta})^2+s^2r^2$.

Hence, the boundary term exactly cancels the divergent contributions of the bulk (corresponding to the $r\to\infty$ limit), so that the total action (5.1) is finite and given by

$$I_{trans} = \frac{\beta n}{G}r_+\int_0^{r_+} dx(1+x^2)^{n-1} - \frac{\beta}{2G}(\sigma^n-\bar{\sigma}^n)\ . \qquad (5.4)$$

The entropy is then given by

$$S = [1-\beta\frac{\partial}{\partial\beta}]I_{trans} = \frac{2\pi n}{G}\int_0^{r_+} dx(1+x^2)^{n-1}\ , \qquad (5.5)$$

which doest not depend on the choice of reference configuration in agreement with previous calculations done by other methods [45, 39, 31]. The energy is

$$E = -\frac{\partial I_{trans}}{\partial\beta} = \frac{1}{2G}(\sigma^n-\bar{\sigma}^n)\ , \qquad (5.6)$$

which depends on the choice of reference background. Note that if the reference configuration is taken to be the black hole vacuum ($\bar{\sigma}=1$) these results agree with the ones of Ref. [31], where AdS spacetime can be regarded to have a nonvanishing "Casimir" energy given by $E_{AdS}=-(2G)^{-1}$.

B. Other topologies ($\gamma=0,-1$)

The evaluation of the euclidean action for black holes with nontrivial topology follows the same steps as in the previous case. The background configuration is the one with $r_+=0$ which corresponds to choose $\bar{\sigma}=\gamma$. Again, the divergent part of the bulk contributions cancel the boundary term, and now the action becomes

$$I_{trans} = \frac{\beta n}{G}\frac{\Sigma_{d-2}}{\Omega_{d-2}}r_+\int_0^{r_+} dx(\gamma+xr^2)^{n-1} - \frac{\beta}{2G}\frac{\Sigma_{d-2}}{\Omega_{d-2}}(\sigma^n-\gamma^n)\ , \qquad (5.7)$$

where Σ_{d-2} stands for the volume of the $d-2$ dimensional base manifold. The entropy now reads

$$S = \frac{2\pi n}{G}\frac{\Sigma_{d-2}}{\Omega_{d-2}}\int_0^{r_+} dx(\gamma+x^2)^{n-1}\ , \qquad (5.8)$$

and the energy is given by

$$E = \frac{\Sigma_{d-2}}{\Omega_{d-2}}(\sigma^n - \gamma^n) \ . \tag{5.9}$$

Note as in the background independent approach, the energy of the configuration with negative constant curvature ($\sigma = 0$) depends on the topology and is given by $E = \gamma^n(2G)^{-1}$ in agreement with [31].

Note that the background substraction procedure that occurs here is not the same as the one proposed by Gibbons and Hawking [47], or by Hawking and Page [48]. In those papers, the actions for two different configurations (for instance, for a massive black hole and Minkowski space) are subtracted, with the additional condition that the metrics match at a very large finite radius r_0 (eventually taken to infinity). That implies two different Euclidean time intervals β and $\overline{\beta}$. Although $\overline{\beta} \to \beta$ when $r_0 \to \infty$, there is an extra contribution to the bulk action coming from the difference of the β's. In our approach there is always only one β, as it must be in order to integrate the boundary term B_{2n}, where both sets of vielbein and spin connections appear entangled, and we have an extra contribution coming from that boundary term. The boundary terms obtained in the standard way coincide with our approach for 2+1 dimensions only.

5.2 Noether charges for AdS gravity

In this section, we show that the energy found from the thermodynamic analysis discussed above agrees with the computation from direct application of Noether's theorem.

5.2.1 Black hole mass from the asymptotic timelike isometry

The Noether charge associated to isometries generated by a vector ξ^μ is

$$Q(\xi) = n(n+1)\int_{\partial\Sigma}\int_0^1 dt < \Delta A F_t^{n-1} I_\xi A_t > \ . \tag{5.10}$$

Here A and $\overline{A}$ correspond to two arbitrary configurations of the form (5.2) for the same topology at the boundary, i. e., for the same γ. For the timelike Killing vector $\xi = \partial\ /\partial t$, (5.10) gives (see Appendix D for the details)

$$Q\left(\frac{\partial}{\partial t}\right) = \frac{\Sigma_{d-2}}{\Omega_{d-2}}(\sigma^n - \bar{\sigma}^n) = E - \bar{E} \ , \tag{5.11}$$

where E stands for the energy computed from thermodynamics. Thus, if $\overline{A}$ is chosen as a reference background solution of the previous section, this charge reproduces the energy computed above. This expression also coincides with the result obtained from the background independent approach in Ref. [31] when the background configuration is the one for which the horizon radius vanishes, i.e., choosing $\bar{\sigma} = \gamma$.

5.2.2 Black hole mass from the gauge Noether charge

Alternatively, the energy can be obtained from the Noether charge associated to gauge transformations, which is given by

$$Q(\lambda) = n(n+1)\int_{\partial\Sigma}\int_0^1 dt < \Delta A F_t^{n-1}\lambda > \ . \tag{5.12}$$

The charge is then evaluated taking both A and $\overline{A}$ as two arbitrary solutions of the the form (5.2), with the same topology at the boundary, for an asymptotically covariantly constant gauge parameter $\lambda = \lambda^a P_a + \frac{1}{2}\lambda^{ab}J_{ab}$ satisfying $\delta_\lambda A = D\lambda = d\lambda + A\lambda - \lambda A = 0$ for $r \to \infty$.

The identity $\mathcal{L}_\xi A = D(I_\xi A) + I_\xi F$, allows to identify the Lie algebra valued parameter λ with a Killing vector as $\lambda = I_\xi A$ provided the curvature vanishes sufficiently fast at infinity. Thus, choosing the gauge parameter as

$$\lambda = I_\xi A_{r\to\infty} \to rP_0 + rJ_{01} \ . \tag{5.13}$$

where ξ is the timelike Killing vector $\partial\ /\partial t$ allows to obtain the the difference between the energies from (5.12)

$$Q\left(\lambda\right) = \frac{\Sigma_{d-2}}{\Omega_{d-2}}(\sigma^n - \bar{\sigma}^n) = E - \bar{E} \ .$$

In the spherically symmetric case, black hole solutions possess $d-1$ independent solutions depending on the arbitrary constants C^1, C^m, with $m = 2, ..., d-1$, given by

$$\lambda^1 = \lambda^{0m} = 0 \tag{5.14}$$

$$\lambda^0 = \lambda^{01} = C^1 r \tag{5.15}$$

$$\lambda^m = \lambda^{m1} = C^m r \tag{5.16}$$

$$\lambda_n^m \tilde{e}^n = \omega_n^m C^n \ , \tag{5.17}$$

so that (5.13) is recovered choosing the parameters as $C^1 = 1$ and $C^m = 0$.

6. Discussion and Comments

The results reported here (and in ref.[31]) support the conjecture that the boundary terms dictated by gauge invariance, supplemented by boundary conditions that make the action principle well defined, give the right conserved charges and black hole entropy without requiring any regularization. The similar problem of computing the conserved charges for the Lovelock theories of gravity in even dimensions was studied in ref.[49], where it was shown that it is possible to regularize the action and the charges by adding a surface term whose exterior derivative is the Euler density of the spacetime manifold.

As we mentioned, $\overline{A}$ could be regarded as a fixed reference or background configuration, a non dynamical entity. However, a better option is to assume both A and $\overline{A}$ in different manifolds with a common boundary. The calculation of the entropy with $\overline{A}$ corresponding to AdS or to a zero mass black hole supports this option. About the question of what is dynamical and what isn't it is worthwhile to remember the comment by Yang and Mills in their landmark paper on non abelian gauge fields about the need to give dynamical content to the gauge field, which they called B_μ [50].

It would be interesting to explore the application of the ideas presented for CS gravity in the presence of "exotic" parity-violating terms [8, 51, 10], as well as for its supersymmetric extensions, and in the framework of the AdS/CFT correspondence [52].

The lesson from this discussion is that the action (4.1), inspired by the transgression form, has an important advantage over the pure CS action. The boundary term incorporated in this way renders the action principle well defined and the Euclidean action for black holes finite, whereas the pure CS action diverges. Hence, it is natural to think of the boundary terms as regulator for the CS theory. It is somehow surprising that these difficulties are solved in one stroke just by requiring strict gauge invariance, and this also suggests that the action principle defined by (4.1), would be a better starting point for a path integral quantization.

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APPENDICES

A. Invariant polynomials, transgression forms and CS forms

In this appendix we review for completeness some elements of the theory of fiber bundles used in this paper. This material can be found in refs.[24, 25, 26, 27, 29, 53]. We are not aware of any reference for the explicit formula for the variation of the transgression that we used (though it is probably known) so we give a derivation in the last subsection of this appendix.

An *invariant polynomial* $P(F)$ is defined as the formal sum

$$P(F) = \sum_{n=0}^{N} \alpha_n < F^{n+1} > \tag{A.1}$$

where

$$< T_{I_1} \ldots T_{I_{n+1}} >= g_{I_1 \cdots I_{n+1}}$$

corresponds to a *symmetric invariant trace* in the algebra of G. This is equivalent to say that $g_{I_1 \cdots I_{n+1}}$ is an invariant symmetric tensor in the algebra of G, which by construction has its indices in the adjoint representation of G.

It can be shown that the invariant polynomials are closed

$$dP(F) = 0$$

therefore locally exact

$$P(F) = d\mathcal{C}_{2n+1}(A, F)$$

where we introduced the CS form defined by

$$\mathcal{C}_{2n+1}(A, F) \equiv (n+1) \int_0^1 ds \quad < AF_s^n >$$

with $A_s = sA$ and $F_s = dA_s + A_s^2$.

A similar relation which holds globally is the *transgression formula*, involving two potentials A and $\overline{A}$ in the same fiber, with curvatures F and $\overline{F}$ respectively

$$< F^{n+1} > - < \overline{F}^{n+1} >= d\mathcal{T}_{2n+1}(A, \overline{A})$$

with the *transgression form* defined as

$$\mathcal{T}_{2n+1}(A, \overline{A}) \equiv (n+1) \int_0^1 dt \quad < (A - \overline{A})F_t^n >$$

with $A_t = tA + (1-t)\overline{A}$ and $F_t = dA_t + A_t^2$.

The transgression form is invariant under gauge transformations for which A and $\overline{A}$ transform with the same group element g of the group G, due to the covariance of $\Delta A \equiv A - \overline{A}$, $(\Delta A)^g = g^{-1}(\Delta A)g$, the covariance of F_t, $F_t^g = g^{-1}F_t g$, and the invariance of the symmetrized trace.

A.1 Cartan operator and homotopy formula

Let A_t be the interpolation between two gauge potentials A and $\overline{A}$,

$$A_t = tA + (1-t)\overline{A} \quad , \quad F_t = dA_t + A_t^2 \quad .$$

The *Cartan homotopy operator* k_{01} acts on polynomials $\mathcal{P}(F_t, A_t)$ and is defined as

$$k_{01}\mathcal{P}(F_t, A_t) = \int_0^1 dt \ l_t \mathcal{P}(F_t, A_t) \quad ,$$

where the action of the operator l_t on arbitrary polynomials of A_t and F_t is defined through

$$l_t A_t = 0 \quad , \qquad l_t F_t = A - \overline{A} \equiv \Delta A \quad ,$$

and the convention that l_t acts as an antiderivative $l_t(\Lambda_p \Sigma_q) = (l_t \Lambda_p)\Sigma_q + (-1)^p \Lambda_p (l_t \Sigma_q)$, where Λ_p and Σ_q are p and q-forms (polynomials in A_t and F_t) respectively.

It can be verified the relationship

$$(l_t d + d l_t)\mathcal{P}(F_t, A_t) = \frac{\partial}{\partial t}\mathcal{P}(F_t, A_t)$$

which can be integrated between 0 and 1 in t to obtain the *Cartan homotopy formula*

$$(k_{01} d + d k_{01})\mathcal{P}(F_t, A_t) = \mathcal{P}(F, A) - \mathcal{P}(\overline{F}, \overline{A}) \quad .$$

For $\mathcal{P} =< F^{n+1} >$ we recover the transgression formula. Putting $\mathcal{P} = \mathcal{C}_{2n+1}$ we get

$$\mathcal{T}_{2n+1} = \mathcal{C}_{2n+1}(A, F) - \mathcal{C}_{2n+1}(\overline{A}, \overline{F}) - d[k_{01}\mathcal{C}_{2n+1}]$$

The $2n$ form B_{2n} is defined by

$$B_{2n}(A, F; \overline{A}, \overline{F}) \equiv k_{01}\mathcal{C}_{2n+1}$$

Explicitly

$$B_{2n} = -n(n+1)\int_0^1 ds \int_0^1 dt \; s \;\; < A_t \Delta A \; F_{st}^{n-1} > \tag{A.2}$$

where $F_{st} = sF_t + s(s-1)A_t^2$

A.2 General variation of the transgression

The transgression form is

$$\mathcal{T}_{2n+1} = (n+1)\int_0^1 dt < \Delta A F_t^n >$$

with $\Delta A = A - \overline{A}$. furthermore

$$A_t = t\Delta A + \overline{A} = tA + (1-t)\overline{A}$$

and

$$F_t = dA_t + A_t^2 = \overline{F} + t\overline{D}(\Delta A) + t^2(\Delta A)^2$$

with $\overline{F} = d\overline{A} + \overline{A}^2$ and $\overline{D}(\Delta A) = d(\Delta A) + \overline{A}(\Delta A) + (\Delta A)\overline{A}$. Notice that the derivative of F_t with respect to the parameter t satisfy

$$\frac{d}{dt}F_t = D_t(\Delta A) = d(\Delta A) + A_t(\Delta A) + (\Delta A)A_t = d(\Delta A) + 2t(\Delta A)^2 + \overline{A}(\Delta A) + (\Delta A)\overline{A}$$

For the general variation of the transgression form we have

$$\delta\mathcal{T}_{2n+1} = (n+1)\int_0^1 dt\{< F_t^n \delta(\Delta A) > + < n(\Delta A)F_t^{n-1}D_t[\delta A_t] >\}$$

but, inside the bracket,

$$D_t[\Delta A F_t^{n-1}\delta A_t] = D_t(\Delta A)F_t^{n-1}\delta A_t - \Delta A F_t^{n-1}D_t[\delta A_t] = \frac{d}{dt}F_t F_t^{n-1}\delta A_t - \Delta A F_t^{n-1}D_t[\delta A_t]$$

and using $\delta A_t = t\delta(\Delta A) + \delta\overline{A}$ we get

$$\delta\mathcal{T}_{2n+1} = (n+1)\int_0^1 dt\{< [F_t^n + tn\frac{d}{dt}F_t F_t^{n-1}]\delta(\Delta A) > + < n\frac{d}{dt}F_t F_t^{n-1}\delta\overline{A} >\}$$
$$-n(n+1)\ d\ \int_0^1 dt < \Delta A F_t^{n-1}\delta A_t >$$

but , when inside the bracket, $F_t^n + tn\frac{d}{dt}F_t F_t^{n-1} = \frac{d}{dt}[tF_t^n]$ and $n\frac{d}{dt}F_t F_t^{n-1} = \frac{d}{dt}F_t^n$, which allow to evaluate explicitly the first to integrals in t giving

$$\delta\mathcal{T}_{2n+1} = (n+1) < F^n\delta(\Delta A) > + (n+1) < (F^n - \overline{F}^n)\delta\overline{A} > - n(n+1)\ d\ \int_0^1 dt < \Delta A F_t^{n-1}\delta A_t >$$

and finally we get for generic infinitesimal variations of the transgressions

$$\delta\mathcal{T}_{2n+1} = (n+1) < F^n\delta A > - (n+1) < \overline{F}^n\delta\overline{A} > - n(n+1)\ d\ \int_0^1 dt < \Delta A F_t^{n-1}\delta A_t >$$

B. Noether's theorem and Conserved Charges

B.1 Noether's Theorem

The variation of differential forms under diffeomorphisms for which the coordinates change as $\delta x^\mu = \xi^\mu$ is given by

$$\delta\alpha(x) = \alpha'(x) - \alpha(x) = -\mathcal{L}_\xi\alpha$$

where $\mathcal{L}_\xi$ is the Lie derivative, which for differential forms can be written as

$$\mathcal{L}_\xi\alpha = [dI_\xi + I_\xi d]\alpha$$

with d the exterior derivative and the contraction operator given by

$$I_\xi\alpha_p = \frac{1}{(p-1)!}\xi^\nu\alpha_{\nu\mu_1\ldots\mu_{p-1}}dx^{\mu_1}\ldots dx^{\mu_{p-1}}$$

The operator I_ξ is an antiderivation, in the sense that acting on the exterior product of two differential forms α_p and β_q of orders p and q it gives $I_\xi(\alpha_p\beta_q) = I_\xi\alpha_p\beta_q + (-1)^p\alpha_p I_\xi\beta_q$. A useful result is that the Lie derivative acting on gauge potentials is

$$\mathcal{L}_\xi A = D(I_\xi A) + I_\xi F$$

where D is the covariant derivative and F the field tensor.
One consider a lagrangian density given by a differential form $L(\phi, \partial\phi)$, where ϕ represents all the dynamical fields. The variation of the lagrangian under diffeomorphisms is given by $\delta L = -d(I_\xi L)$, as $dL = 0$ because the order of L is equal to the dimension of the space. One considers a class of transformations under which the lagrangian is quasi-invariant, combined with diffeomorphisms. Under these transformations the variation of the lagrangian is

$$\delta L = d\Omega - d(I_\xi L)$$

where the first total derivative come from the transformations considered and the second one from the diffeomorphisms.

On the other hand, the standard procedure leading to the equations of motion gives the variation of the lagrangian as the equations of motion times the variation of ϕ plus a boundary term

$$\delta L = (E.d.M.)\delta\phi + d\Theta$$

where the variations $\delta\phi$ are infinitesimal but arbitrary in form. ¿From this two expressions of the variation, assuming that the variations in both are restricted to transformations of the class considered in the first expression of δL and equating, that if the E.O.M. hold

$$d[\Omega - I_\xi L - \Theta] = 0$$

It follows that the so called Noether current

$$\star j = \Omega - I_\xi L - \Theta$$

is conserved $d(\star j) = 0$.

In the next two subsections we will deduce the general form of the gauge and diffeomorphism Noether charges for Transgression and Chern-Simons theories. In the calculation of the expressions for the charges both gauge fields appearing in the transgression are varied. If one prefers to consider the second field as non dynamical, one could think of varying it as a trick, analogous to varying the flat Minkowski metric to compute the energy momentum tensor of a field in flat space-time, for a theory for which the metric is non dynamical.

B.2 Diffeomorphism Noether charges

The variation of the gauge potentials under diffeomorphisms is

$$\delta_\xi A = -\mathcal{L}_\xi A = -D[I_\xi A] - I_\xi F = -[I_\xi d + dI_\xi]A \tag{B.1}$$

$$\delta_\xi \overline{A} = -\mathcal{L}_\xi \overline{A} = -\overline{D}[I_\xi \bar{A}] - I_\xi \bar{F} = -[I_\xi d + dI_\xi]\overline{A} \tag{B.2}$$

$$\delta_\xi A_t = -\mathcal{L}_\xi A_t = -D_t[I_\xi A_t] - I_\xi F_t = -[I_\xi d + dI_\xi]A_t \tag{B.3}$$

Inserting this in the variation of the transgression

$$\delta\mathcal{T}_{2n+1} = (n+1) < F^n \delta A > -(n+1) < \bar{F}^n \delta \bar{A} > -n(n+1)\ d\int_0^1 dt < \Delta A F_t^{n-1} \delta A_t >$$

we can read the form Θ that appears in the Noether theorem

$$\Theta = -n(n+1)\int_0^1 dt < \Delta A F_t^{n-1} \delta_\xi A_t > \tag{B.4}$$

or

$$\Theta = n(n+1)\int_0^1 dt < \Delta A F_t^{n-1} D_t[I_\xi A_t] + \Delta A F_t^{n-1} I_\xi F_t > \tag{B.5}$$

But, inside the bracket,

$$D_t[\Delta A F_t^{n-1} I_\xi A_t] = D_t \Delta A F_t^{n-1} I_\xi A_t - \Delta A F_t^{n-1} D_t[I_\xi A_t] = \frac{d}{dt} F_t F_t^{n-1} I_\xi A_t - \Delta A F_t^{n-1} D_t[I_\xi A_t]$$

then

$$\Theta = n(n+1)\int_0^1 dt < \frac{d}{dt} F_t F_t^{n-1} I_\xi A_t + \Delta A F_t^{n-1} I_\xi F_t > -n(n+1)\ d\int_0^1 dt < \Delta A F_t^{n-1} I_\xi A_t > \tag{B.6}$$

For the term $I_\xi L$ in the Noether current we have

$$I_\xi L = I_\xi \mathcal{T}_{2n+1} = (n+1)\int_0^1 dt < I_\xi(\Delta A) F_t^n - n \Delta A F_t^{n-1} I_\xi F_t > \tag{B.7}$$

The current is $*j = \Omega - [\Theta + I_\xi L]$, but $\Omega = 0$ because the action is invariant under diffeomorphisms, then

$$*j = -[\Theta + I_\xi L] = -(n+1)\int_0^1 dt < n\frac{d}{dt} F_t F_t^{n-1} I_\xi A_t + I_\xi \Delta A F_t^n >$$

$$+n(n+1)\ d\int_0^1 dt < \Delta A F_t^{n-1} I_\xi A_t >$$

but $I_\xi A_t = t I_\xi(\Delta A) + I_\xi \overline{A}$, then $I_\xi(\Delta A) = \frac{d}{dt} I_\xi A_t$ and therefore

$$< n\frac{d}{dt} F_t F_t^{n-1} I_\xi A_t + I_\xi \Delta A F_t^n > = \frac{d}{dt} < F_t^n I_\xi A_t >$$

This expression allows to integrate the first terms of the current,yielding

$$*j = < F^n I_\xi A > - < \bar{F}^n I_\xi \overline{A} > +n(n+1)\ d\int_0^1 dt < \Delta A F_t^{n-1} I_\xi A_t > \tag{B.8}$$

The first two terms of the second member vanish due to the E.O.M., then

$$*j = \ d\mathbf{Q}_\xi \tag{B.9}$$

with

$$\mathbf{Q}_\xi = +n(n+1)\int_0^1 dt < \Delta A F_t^{n-1} I_\xi A_t > \tag{B.10}$$

The conserved charge is then

$$Q(\xi) = \int_{\partial\Sigma} \mathbf{Q}_\xi = +n(n+1)\int_{\partial\Sigma}\int_0^1 dt < \Delta A F_t^{n-1} I_\xi A_t > \tag{B.11}$$

¿From this expression one gets the one for pure Chern-Simons by setting $\overline{A} = 0$, because the configuration $\overline{A} = 0$ satisfies the E.O.M..

B.3 Gauge Noether charges

The variation of A and $\overline{A}$ under gauge transformations is

$$\delta_\lambda A = -D\lambda \quad , \quad \delta_\lambda \overline{A} = -\overline{D}\lambda, \tag{B.12}$$

which implies

$$\delta_\lambda A_t = -D_t\lambda = -d\lambda - A_t\lambda + \lambda A_t \tag{B.13}$$

The E.O.M., which we assume are satisfied by both fields A and $\overline{A}$, are $< F^n T_I >= 0$ and $< \bar{F}^n T_I >= 0$. It follows that we can read the form Θ appearing in the Noether theorem (see appendix) from the expression for the variation

$$\Theta = n(n+1)\int_0^1 dt < \Delta A F_t^{n-1} D_t\lambda > \tag{B.14}$$

The form Ω is zero in this case, because the transgression is gauge invariant. It follows that the conserved current is

$$*j_\lambda = -\Theta = -n(n+1)\int_0^1 dt < \Delta A F_t^{n-1} D_t\lambda > \tag{B.15}$$

Furthermore $*j_\lambda = d\mathbf{Q}_\lambda$ with

$$\mathbf{Q}_\lambda = n(n+1)\int_0^1 dt < \Delta A F_t^{n-1}\lambda > \tag{B.16}$$

because

$$d\mathbf{Q}_\lambda = n(n+1)\int_0^1 dt < D_t[\Delta A F_t^{n-1}\lambda] > \tag{B.17}$$

or

$$d\mathbf{Q}_\lambda = n(n+1)\int_0^1 dt < \frac{d}{dt}F_t F_t^{n-1}\lambda - \Delta A F_t^{n-1} D_t\lambda] > \tag{B.18}$$

and, using $\frac{d}{dt}F_t^{n-1} = \frac{1}{n}\frac{d}{dt}F_t^n$ inside the bracket $< \quad >$ we get

$$d\mathbf{Q}_\lambda = (n+1) < (F_1^n - F_0^n)\lambda > -n(n+1)\int_0^1 dt < \Delta A F_t^{n-1} D_t\lambda > \tag{B.19}$$

where the first term of the second member is zero due to the E.O.M.

The conserved gauge charge is then

$$Q(\lambda) = \int_{\partial\Sigma}\mathbf{Q}_\lambda = n(n+1)\int_{\partial\Sigma}\int_0^1 dt < \Delta A F_t^{n-1}\lambda > \tag{B.20}$$

This expression for $Q(\lambda)$ yields the one for a pure Chern-Simons theory, by setting $\overline{A} = 0$, because the configuration $\overline{A} = 0$ does satisfy the E.O.M., in agreement with the hypotheses of our derivation.

B.4 Algebra of the gauge charges

If

$$\mathbf{Q}_\lambda = n(n+1)\int_0^1 dt < \Delta A F_t^{n-1}\lambda > \tag{B.21}$$

the algebra of the gauge charges is given by

$$\{\mathbf{Q}(\lambda), \mathbf{Q}(\eta)\} := \delta_\eta \mathbf{Q}(\lambda) \tag{B.22}$$

To evaluate this expression we notice that under finite gauge transformations

$$A \to g^{-1}[A+d]g \quad , \quad \overline{A} \to g^{-1}[\overline{A}+d]g$$

hence $\Delta A \to g^{-1}\Delta A\ g$, $A_t \to g^{-1}[A_t+d]g$, $F \to gFg^{-1}$, $\bar{F} \to g\bar{F}g^{-1}$ and $F_t \to gF_tg^{-1}$. Writing g in the form $g = exp[-\lambda]$ with $\lambda = \lambda^I T_I$, in the case of an infinitesimal λ we recover the expressions for infinitesimal gauge transformations $\delta_\lambda A = -D\lambda$, $\delta_\lambda \overline{A} = -\overline{D}\lambda$ and $\delta_\lambda A_t = -D_t\lambda = -d\lambda - A_t\lambda + \lambda A_t$.

To compute δ_η on $\mathbf{Q}(\lambda)$ it is easier to start with a finite transformation $g = exp[-\eta]$ and then take the limit $\eta \ll 1$. We have

$$< \Delta A F_t^{n-1}\lambda > \to < g^{-1}\Delta A F_t^{n-1} g\lambda >$$

and, using the cyclic property of the trace

$$< g^{-1}\Delta A F_t^{n-1} g\lambda > = < \Delta A F_t^{n-1} g\lambda g^{-1} >$$

and using the infinitesimal form $g = 1 - \eta$ it results

$$\delta_\eta \mathbf{Q}(\lambda) = \mathbf{Q}([\lambda, \eta]) \tag{B.23}$$

or

$$\{\mathbf{Q}(\lambda), \mathbf{Q}(\eta)\} = \mathbf{Q}([\lambda, \eta]) \tag{B.24}$$

Notice the absence of central charges in the second member, which was to be expected, because central charges are characteristic of quasi-invariant lagrangians and transgressions are truly invariant.

C. Gravity and the transgression form

In this Appendix we derive the explicit expression for the AdS group transgression form given in the main text.

The AdS curvatures are

$$F = \frac{1}{2}\mathbf{R}J + TP \quad , \quad \overline{F} = \frac{1}{2}\overline{\mathbf{R}}J + \overline{T}P \tag{C.1}$$

where $R = d\omega + \omega^2$, $\overline{R} = d\overline{\omega} + \overline{\omega}^2$ are the curvatures, $T^a = de^a + \omega^a{}_b e^b = De^a$, $\overline{T}^a = d\overline{e}^a + \overline{\omega}^a{}_b \overline{e}^b = \overline{D}\overline{e}^a$ are the torsions, and $\mathbf{R} = R + e^2$, $\overline{\mathbf{R}} = \overline{R} + \overline{e}^2$. Furthermore

$$F_t = \frac{1}{2}\mathbf{R}_t J + T_t P \tag{C.2}$$

with

$$\mathbf{R}_t = t\mathbf{R} + (1-t)\overline{\mathbf{R}} - t(1-t)[\theta^2 + E^2] \tag{C.3}$$

where we define $\theta = \omega - \overline{\omega} = \Delta\omega$ (*even though* θ *is not the Second Fundamental Form for generic* ω *and* $\overline{\omega}$) and $E = e - \overline{e}$. $\mathbf{R}_t$ can also be written as

$$\mathbf{R}_t = tR + (1-t)\overline{R} - t(1-t)\theta^2 + e_t^2 \tag{C.4}$$

where $e_t = te + (1-t)\overline{e}$. Furthermore

$$T_t = tT + (1-t)\overline{T} - t(1-t)((\theta E)) \tag{C.5}$$

where the double parentheses stands for contractions, for instance $((\omega^2))^{ab} \equiv \omega^{af}\omega_f{}^b \equiv (\omega^2)^{ab}$, and $((\omega e)) \equiv \omega^{af} e_f$.

The AdS transgression form must be the of the form

$$\mathcal{T}^{AdS}_{2n+1} = \kappa \int_0^1 dt \epsilon (R + t^2 e^2)^n e - \kappa \int_0^1 dt \epsilon (\overline{R} + t^2 \overline{e}^2)^n \overline{e} - dB_{2n} \tag{C.6}$$

where the boundary term B_{2n} is what we intend to determine.

The variation of the bulk AdS CS form is

$$\delta\mathcal{C}^{AdS}_{2n+1} = \kappa\epsilon\mathbf{R}^n \delta e + \kappa\epsilon n \mathbf{R}^{n-1} T\delta\omega + d\Xi \tag{C.7}$$

where the boundary contribution is

$$\Xi = -\kappa n \int_0^1 dt \epsilon (R + t^2 e^2)^{n-1} e\delta\omega \tag{C.8}$$

The variation of the transgression must then be

$$\delta\mathcal{T}^{AdS}_{2n+1} = \kappa\epsilon\mathbf{R}^n\delta e + \kappa\epsilon n\mathbf{R}^{n-1}T\delta\omega - \kappa\epsilon\overline{\mathbf{R}}^n\delta\overline{e} - \kappa\epsilon n\overline{\mathbf{R}}^{n-1}\overline{T}\delta\overline{\omega} + d\Xi - d\overline{\Xi} + d[\delta B_{2n}] \tag{C.9}$$

But

$$\kappa\epsilon\mathbf{R}^n\delta e + \kappa\epsilon n\mathbf{R}^{n-1}T\delta\omega - \kappa\epsilon\overline{\mathbf{R}}^n\delta\overline{e} - \kappa\epsilon n\overline{\mathbf{R}}^{n-1}\overline{T}\delta\overline{\omega} = \\ = (n+1) < F^n\delta A > -(n+1) < \overline{F}^n\delta\overline{A} > \tag{C.10}$$

So, except for an irrelevant closed form, it must hold that

$$\Theta_{2n} = \Xi - \overline{\Xi} + \delta B_{2n} \tag{C.11}$$

Our goal is to find B_{2n}. To that end we notice that Ξ contains only $\delta\omega$ while $\overline{\Xi}$ contains only $\delta\overline{\omega}$, therefore the coefficients of δe and $\delta\overline{e}$ on Θ_{2n} and B_{2n} must be the same. We will exploit this fact to integrate the variations.

For AdS

$$\Theta_{2n} = -\kappa n \int_0^1 dt \epsilon \{[\theta \mathbf{R}_t^{n-1}]\delta e_t + [E\mathbf{R}_t^{n-1} + (n-1)\theta \mathbf{R}_t^{n-2} T_t]\delta\omega_t\} \tag{C.12}$$

But

$$\epsilon \mathbf{R}_t^{n-1}\delta e_t = \epsilon[\xi_t(\omega,\overline{\omega}) + e_t^2]^{n-1}\delta e_t \tag{C.13}$$

with

$$\xi_t(\omega,\overline{\omega}) = tR + (1-t)\overline{R} - t(1-t)\theta^2 \tag{C.14}$$

Expanding we get

$$\epsilon \mathbf{R}_t^{n-1}\delta e_t = \epsilon \left\{ \sum_{k=0}^{n-1} C_k^{n-1} \xi_t^{n-1-k} e_t^{2k}\delta e_t \right\} = \epsilon \delta_{(e_t)} \left\{ \sum_{k=0}^{n-1} \frac{C_k^{n-1}}{2k+1} \xi_t^{n-1-k} e_t^{2k+1} \right\} \tag{C.15}$$

where we used the symbol $\delta_{(e_t)}$ in the last member to indicate that only the vielbeins are varied there. We can then write

$$\begin{aligned} \epsilon \mathbf{R}_t^{n-1}\delta e_t &= \delta_{(e_t)} \left\{ \int_0^1 ds \epsilon \left[\sum_{k=0}^{n-1} C_k^{n-1} \xi_t^{n-1-k} s^{2k} e_t^{2k} \right] e_t \right\} \\ &= \epsilon \delta_{(e_t)} \left\{ \int_0^1 ds \ \epsilon \theta e_t [\xi_t + s^2 e_t^2]^{n-1} \right\} \end{aligned}$$

where again only the vielbeins are varied. It follows that

$$B_{2n} = \kappa n \int_0^1 dt \int_0^1 ds \ \epsilon \ \theta e_t \left\{ tR + (1-t)\overline{R} - t(1-t)\theta^2 + s^2 e_t^2 \right\}^{n-1} \tag{C.16}$$

It may worry the reader that a contribution to $B_2 n$ depending only on ω and $\overline{\omega}$ could have been missed in this approach, however looking at Θ_{2n} above it is clear that such a contribution does not exist, as every term has a dependence on e or $\overline{e}$.

D. Conserved charges for black holes with a reference background

In order to evaluate the Noether charge associated to the time like Killing vector $\xi = \frac{\partial}{\partial t}$ for two black hole configurations with parameters σ and $\overline{\sigma}$ respectively the relevant non vanishing ingredients [39, 31] are[5]

$$\begin{aligned} \theta^{1m} &= -(\Delta - \overline{\Delta})\tilde{e}^m \ , & (\theta^2)^{mn} &= -(\Delta - \overline{\Delta})\tilde{e}^m \tilde{e}^n \\ I_\xi A_t &= [t\Delta + (1-t)\overline{\Delta}]P_0 + rJ_{01} \ , & (e_t^2)^{mn} &= r^2 \tilde{e}^m \tilde{e}^n \\ R^{mn} &= (1-\Delta^2)\tilde{e}^m\tilde{e}^n \ , & \bar{R}^{mn} &= (1-\overline{\Delta}^2)\tilde{e}^m\tilde{e}^n \\ R^{0m} &= -\Delta r \ dt \tilde{e}^m \ , & \bar{R}^{0m} &= -\overline{\Delta} r \ dt\tilde{e}^m \end{aligned} \tag{D.1}$$

[5]in this Appendix, as in the previous one $\theta = \omega - \bar{\omega}$, even though for generic connections θ is not he Second Fundamental Form. We also use the $\mathbf{R}_t$ notation of the previous Appendix.

Also $\Delta = (1-\sigma+r^2)^{\frac{1}{2}}$ with $\sigma = (2Gm+1)^{\frac{1}{n}}$ and $\overline{\Delta} = (1-\overline{\sigma}+r^2)^{\frac{1}{2}}$ with $\overline{\sigma} = (2G\overline{m}+1)^{\frac{1}{n}}$. In the previous expressions m and $\overline{m}$ are just certain constants of integration of the solutions, which turn to be closely related to the energy coming from the thermodynamics and the one coming from the Noether charge, justifying in retrospect to call those parameters the 'masses' of the black holes.

We will need the components of $\mathbf{R}_t = tR + (1-t)\bar{R} - t(1-t)\theta^2 + e_t^2$, where $e_t = te + (1-t)\bar{e})$, with group indices mn. Those are

$$\left(tR + (1-t)\bar{R} - t(1-t)\theta^2 + e_t^2\right)^{mn} = \left\{1 - [t\Delta + (1-t)\overline{\Delta}]^2 + r^2\right\} \tilde{e}^m \tilde{e}^n \tag{D.2}$$

The charge coming from eq.(2.6)is in this case

$$Q(\frac{\partial}{\partial t}) = \kappa n \int_{\partial\Sigma} \int_0^1 dt \; 2\epsilon_{01m_1m_2\ldots m_{2n-1}} [I_\xi A_t]^0 \theta^{1m_1} \mathbf{R}_t^{m_2m_3} \ldots \mathbf{R}_t^{m_{2n-2}m_{2n-1}} \tag{D.3}$$

where we used that the only non vanishing components of ΔA with support in the spatial boundary are θ^{1m}, so the index 1 is necessarily there, while the index 0 must then be in $I_\xi A_t$, which must therefore contain the only generator P_a. Inserting the expressions for the terms of this equation and integrating in the spatial boundary $\partial\Sigma \equiv S^{d-2}$ we get

$$\begin{aligned} Q(\frac{\partial}{\partial t}) = -\kappa 2n\Omega_{d-2} \int_0^1 dt \{ & (\Delta - \overline{\Delta})[t\Delta + (1-t)\overline{\Delta}] \times \\ & \times [1 - (t\Delta + (1-t)\overline{\Delta})^2 + r^2]^{n-1} \} \end{aligned} \tag{D.4}$$

where Ω_{d-2} is the volume of the sphere of dimension $d-2$ resulting from the integration of the angular variables. The integral in the parameter t can be done trough the substitution $u = 1 - (t\Delta + (1-t)\overline{\Delta})^2 + r^2$ and the result is

$$Q(\frac{\partial}{\partial t}) = \kappa(d-2)!\Omega_{d-2}[u^n] \mid_{1-\overline{\Delta}^2+r^2}^{1-\Delta^2+r^2} \tag{D.5}$$

Notice that $1 - \Delta^2 + r^2 = \sigma$ and $1 - \overline{\Delta}^2 + r^2 = \overline{\sigma}$. The result is

$$Q(\frac{\partial}{\partial t}) = \kappa(d-2)!\Omega_{d-2}[\sigma^n - \overline{\sigma}^n] \tag{D.6}$$

Using the expressions for σ and $\overline{\sigma}$, that $\kappa = \frac{1}{2G(d-2)!\Omega_{d-2}}$ we get

$$Q(\frac{\partial}{\partial t}) = m - \overline{m} = E - \bar{E} \tag{D.7}$$

Particular cases of this expression are the zero mass black hole (with $\overline{m} = 0$) and AdS (with $\overline{m} = -\frac{1}{2G}$). The AdS mass can be thought as a vacuum or Casimir energy.

A similar calculation yields the charge for topological black holes. Topological black holes are labeled by the parameter γ which can take the values 1, 0 or -1. The line element is

$$ds^2 = -\Delta^2 dt^2 + \frac{1}{\Delta^2}dr^2 + r^2 d\Sigma^2_{d-2} \tag{D.8}$$

where $d\Sigma^2_{d-2}$ is the line element of the $d-2$ sphere ($\gamma = 1$, the case just considered), plane ($\gamma = 0$) or hyperboloid ($\gamma = -1$) and

$$\begin{aligned} \Delta^2 &= \gamma - \sigma + r^2 \\ \sigma &= (2G\mu + \delta_{1,\gamma})^{\frac{1}{n}} \end{aligned} \tag{D.9}$$

where Σ_{d-2} stands for the volume of the corresponding $d-2$ dimensional manifold and μ is a parameter or integration constant which, after the thermodynamics and Noether charge computation of the energy, will be regarded as a energy or mass density.

The calculation of Noether charge is analogous. We now have

$$(\mathbf{R}_t)^{mn} = \left\{\gamma - [t\Delta + (1-t)\overline{\Delta}]^2 + r^2\right\} \tilde{e}^m \tilde{e}^n \tag{D.10}$$

The charge is

$$Q(\frac{\partial}{\partial t}) = \kappa(d-2)!\Sigma_{d-2}[u^n] \mid^{\sigma}_{\overline{\sigma}} = \kappa(d-2)!\Sigma_{d-2}[\sigma^n - \overline{\sigma}^n] \tag{D.11}$$

where now $\partial\Sigma \equiv \Sigma^{d-2}$. Using the expressions for σ and $\overline{\sigma}$ and $\kappa = \frac{1}{2G(d-2)!\Omega_{d-2}}$ we get

$$Q(\frac{\partial}{\partial t}) = \frac{\Sigma_{d-2}}{\Omega_{d-2}}[\mu - \overline{\mu}] = E - \bar{E} \tag{D.12}$$

which implies that the parameter μ is a sort of energy density.

We will furthermore consider the $\overline{A}$ configuration for with the horizon radius is zero $\overline{r}_+ = 0$, so that $\overline{\sigma} = \gamma$. For $\gamma = 0, -1$ (the $\gamma = 1$ case was studied above) we get

$$Q(\frac{\partial}{\partial t}) = \frac{\Sigma_{d-2}}{\Omega_{d-2}}(\mu - \frac{\gamma^n}{2G}) \tag{D.13}$$

Here $\frac{\gamma^n}{2G}$ can be interpreted as a vacuum or Casimir energy density.

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